



**AFRL-RB-WP-TM-2011-3043**

## **RISK-BASED COMPUTATIONAL PROTOTYPING (BRIEFING CHARTS)**

**Philip Beran, José Camberos, Ned Lindsley, and Bret Stanford**

**Multi-Disciplinary Technologies Branch  
Structures Division**

**OCTOBER 2010  
Interim Report**

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14. ABSTRACT We are developing computational methods that will enable the computational design of air vehicles accounting for inherently nonlinear dynamic behaviors. These behaviors fall into two categories: behaviors that are beneficial for vehicle operation, such as could be observed for micro air vehicles propelled by wing flapping (e.g., a productive energy transfer between the unsteady vortical flow produced by a flapping wing and the associated nonlinear deformation of the wing), and behaviors that constrain vehicle operation, such as in the dangerous limit-cycle oscillation of large aircraft. In either case, the design space is large and the analysis multi-disciplinary. We have investigated different ways of computing sensitivities of vehicle dynamics to a large number of design variables, compressing the computation using model reduction, and assessing the impact of variability on the reliability of the system.					
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# Risk-Based Computational Prototyping

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Dr. Ned Lindsley, Aerospace Engineer, Co-PI

Dr. Bret Stanford, Post-Doctoral Research Associate

*Multidisciplinary Science & Technology Center (MSTC)*

Air Vehicles Directorate

September, 2010



# MSTC Organization & Activity



**Mission: Integrate multiple disciplines to *discover and exploit new phenomena* for system *optimization and assessment* of revolutionary aerospace vehicles**



Center Director

Branch Chief  
- Tech Advisor

## Prototype Representation & Design Exploration Methods

- Parametric Geometry & Mesh
- Subsystem Representation
- Design Space Exploration & Optimization
- Risk-based Design

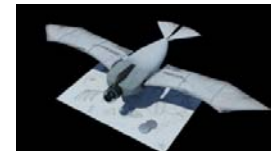
## Analysis Methods for Prototypes

- Multidisciplinary Analysis
- Appropriate-fidelity Solutions and Sensitivities
- Nondeterministic Models

## Prototype Validation & Assessment

- HiFi QTA
- Prototype Experimental Validation
- TRL Assessment

Shared Activity - Utilize a Unified Framework (SORCER, MODEL Center)



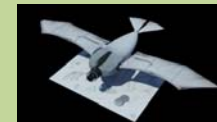


# Some Significant Collaborations



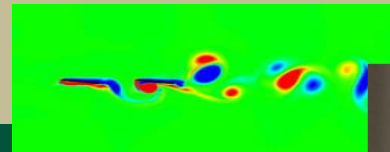
MSTC Collaborative Center with  
VPI & SU, WSU, and University of Maryland  
(Formed March 2009)

Prof Kapania, Director  
Dr. Kolonay, PM



AFRL/RB and WSU Center for Micro Air  
Vehicle Studies (Formed June 2010)

Prof Huang, Director  
Dr. Beran, PM

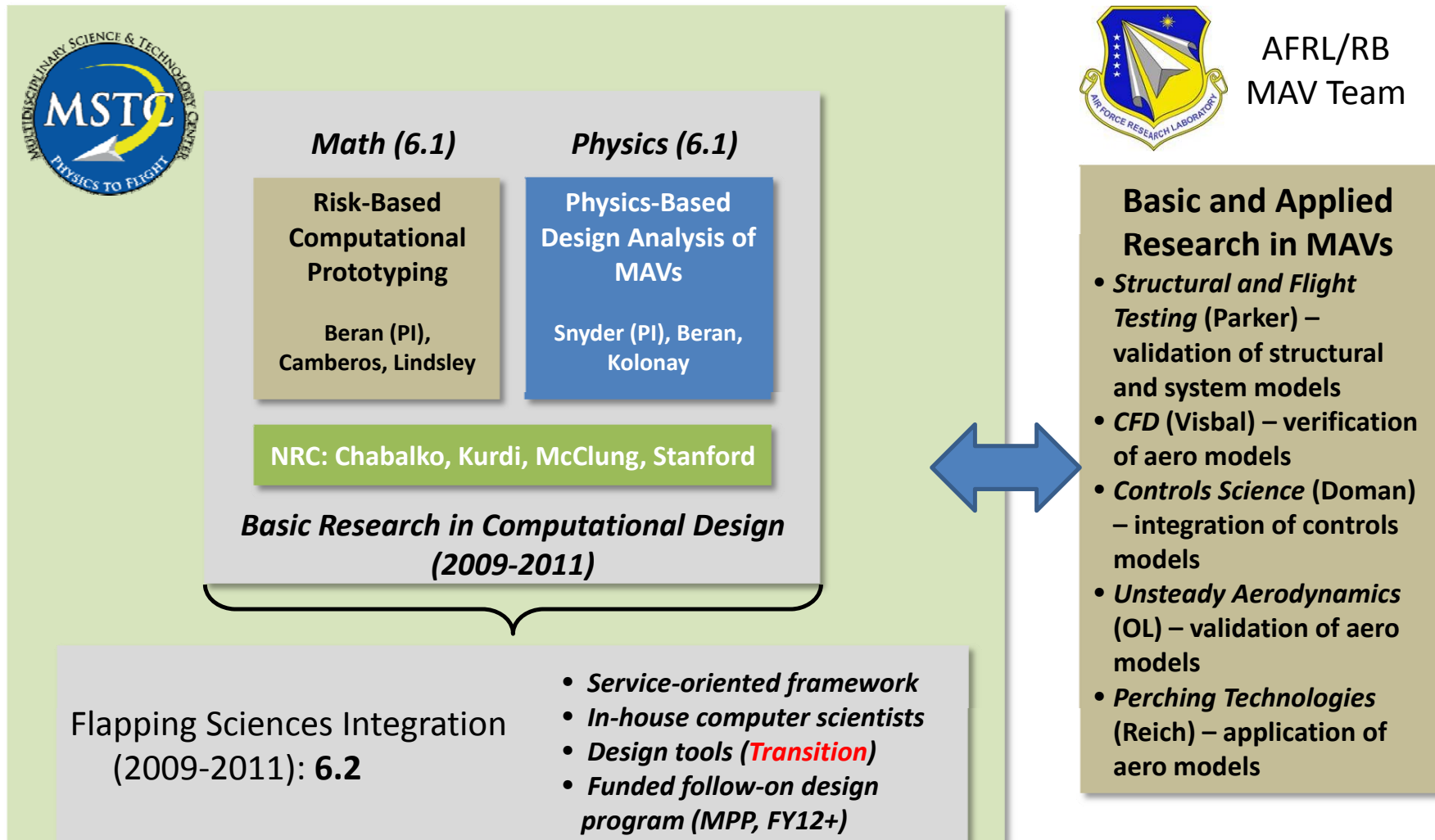


- Prof Missoum, Mr. Basudhar (UA, Tucson) and Dr. Lambe (MSSRC) – RBDO with LCO
- Prof Dong and Mr. Gaston (WSU) – ROM and Simulation of falling bodies
- Prof McFarland and Mr. Hubbard (UIUC) – Transmission design with nonlinearity





# Internal Collaborations in MAVs





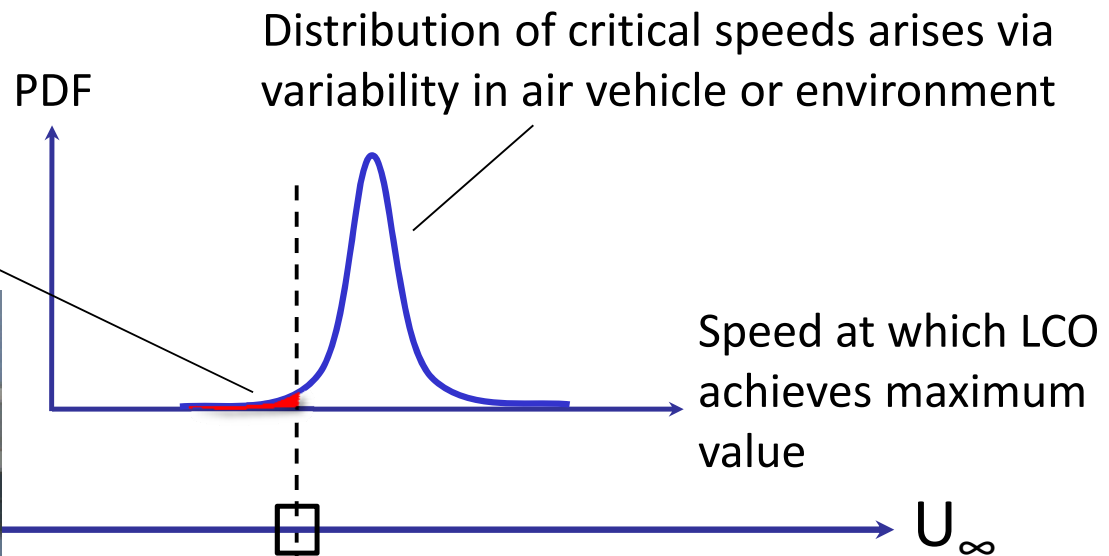
# Role of Computational Mathematics



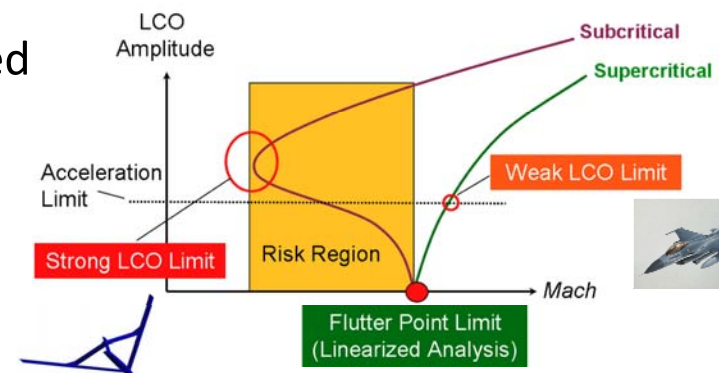
$P_F$  = Probability of Failure (large LCO)  
*Must be sufficiently small*



*Large Vehicles*



Target Flight Speed



Computational mathematics needed for physics-based design of reliable vehicles

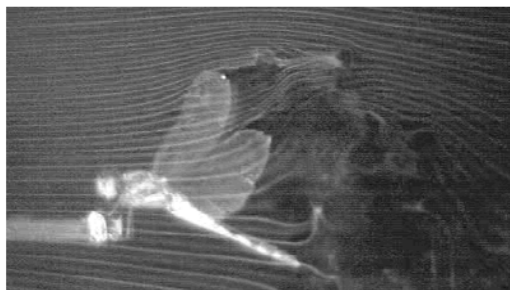




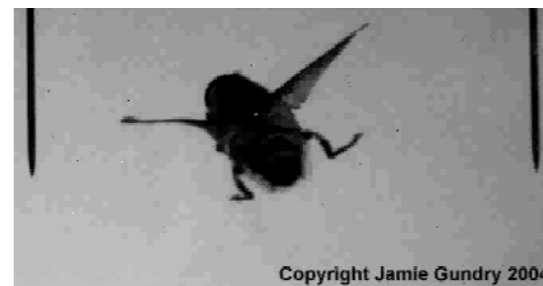
## Role of Computational Mathematics (cont.)



- Exploit nonlinear aeroelastic interactions for small aircraft



Unsteady Flow, Iida (2004)



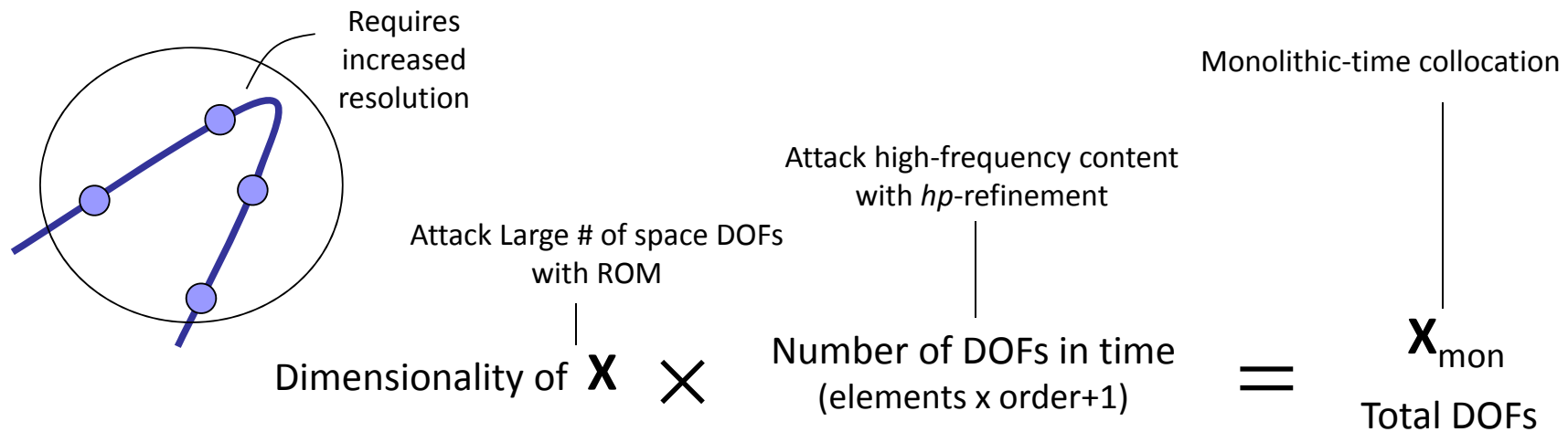
Unsteady Deformations

- Numerous challenges for design of Micro air vehicles (MAVs)
  - Physics Rich (must be a physics-based approach)
  - Complex and time-dependent actuations (unsteady)
  - Non-conventional geometries and structural topologies
  - Power-based integration of propulsion, structure, control components

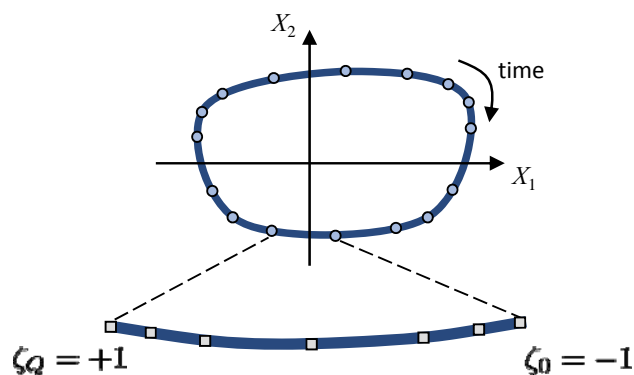
Computational mathematics needed for physics-based design of MAVs



# Spectral Formulation for Time-Periodic Systems



- Uses a local basis instead of global basis



$$X_e(\zeta) = \sum_{q=0}^m X_e(\zeta_q) \Psi_q(\zeta)$$

- $m$  – Order of the spectral element
- $\zeta_q$  – Zeroes of the Lobatto-Legendre polynomials
- $\Psi_q(\zeta)$  – Lagrange polynomial of order  $m$

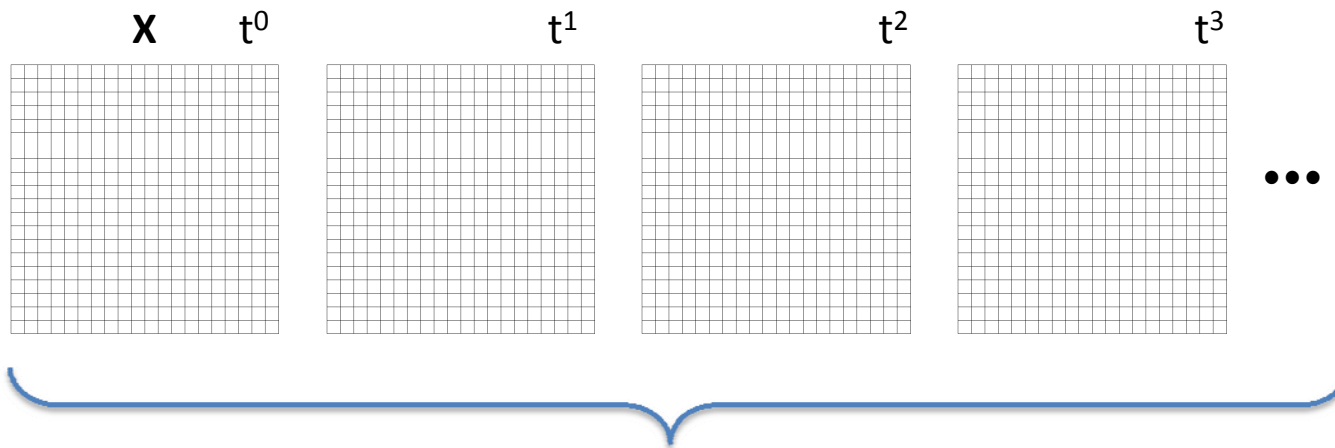
Kurdi and Beran, "Spectral Element Method in Time for Rapidly Actuated Systems," *JCP*, Vol. 227, No. 3, 2008, pp. 1809-1835.



# Monolithic-Time Collocation



Arrays corresponding to a  
discrete 2D field variable



$$\mathbf{X}_{\text{mon}} = [X^0, X^1, X^2, X^3, \dots]$$

Context for time-periodic *and* transient solutions



# Adjoint-Variable Approach

1

Solve  $\mathbf{F}_{\text{mon}}(\mathbf{X}_{\text{mon}}, \lambda) = 0$

$H(\mathbf{X}_{\text{mon}})$  = objective

$\mathbf{F}_{\text{mon}}$  = equation residual

Sensitivity:

$$\frac{dH}{d\lambda} = - \overbrace{\frac{\partial H}{\partial \mathbf{X}_{\text{mon}}} \left( \frac{\partial \mathbf{F}_{\text{mon}}}{\partial \mathbf{X}_{\text{mon}}} \right)^{-1}}^{\text{adjoint}} \underbrace{\frac{\partial \mathbf{F}_{\text{mon}}}{\partial \lambda}}_{\text{direct}}$$

2

$$\left( \frac{\partial \mathbf{F}_{\text{mon}}}{\partial \mathbf{X}_{\text{mon}}} \right)^T \mathbf{C}_{\text{mon}} = \left( \frac{\partial H}{\partial \mathbf{X}_{\text{mon}}} \right)^T$$

High cost:  
computed once

3

$$\frac{dH}{d\lambda} = -\mathbf{C}_{\text{mon}}^T \frac{\partial \mathbf{F}_{\text{mon}}}{\partial \lambda}$$

Inexpensive:  
analytic or finite-difference (repeat for each  
variable) about monolithic solution

Goal: Examine challenge of storing  $\mathbf{X}_{\text{mon}}$  between step 1 and 2



# Adjoint Computation for Transient Sensitivity Analysis



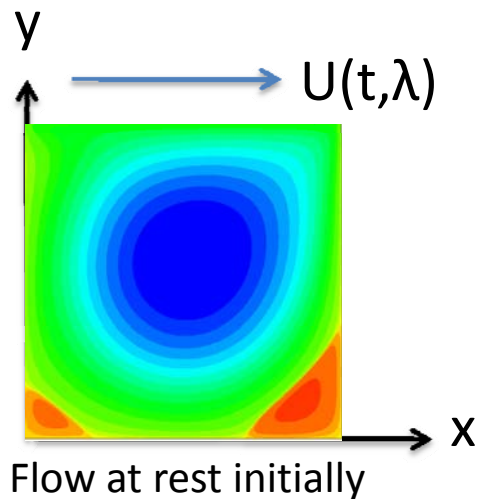
Goal: Develop a sensitivity analysis process that scales well with total # DOFs

- Interested in the adjoint-variable approach in anticipation of:
  - many design variables (not true of direct and sampling based approaches)
  - use of gradient-based optimization (trade global effectiveness for efficiency)
- Some relevant literature
  - Nadarajah and Jameson, "Optimum Shape Design for Unsteady Flows with Time-Accurate Continuous and Discrete Adjoint Methods," AIAA Journal Vol. 45, No. 7, 2007
  - Thomas, Hall, and Dowell, "A Discrete Adjoint Approach for Modeling Unsteady Aerodynamic Design Sensitivities," AIAA 2003-0041, 2003
  - Mani and Mavriplis, "An Unsteady Discrete Adjoint Formulation for Two-Dimensional Flow Problems with Deforming Meshes," AIAA 2007-60, 2007
- Create a sample problem to explore a POD-based approach to eliminate challenge of storing the forward solution



# Problem Description

Transient analysis of incompressible flow in a square cavity with unsteady lid



- Steady:  $U = 1$  (impulsive)
  - verify; assess accuracy
- Transient:  $U = \frac{1}{2}(1 - \cos(f t))$ 
  - define  $H$ , a function of the transient solution
  - compute sensitivity of  $H$  to frequency,  $f$
- Streamfunction-vorticity form

$$\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \frac{1}{\text{Re}} \nabla^2 \omega \quad \nabla^2 \Psi = -\omega \quad u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x}$$



# Discretization and Time Integration



Explicit/implicit formulation

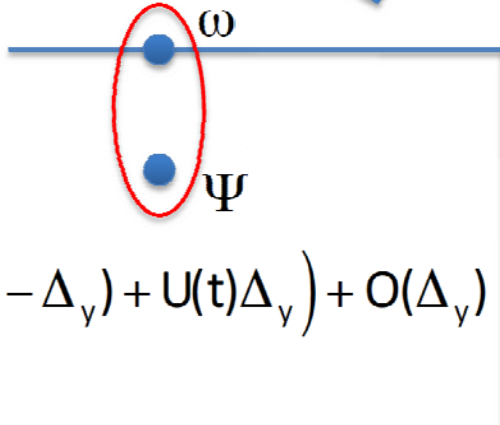
1 
$$\frac{\omega^{n+1} - \omega^n}{dt} + \left( u\delta_x \omega + v\delta_y \omega \right)^n = \frac{1}{\text{Re}} \left( \delta_{xx} + \delta_{yy} \right) \omega^{n+1}$$

2<sup>nd</sup>-order-accurate, central-difference operators

2 
$$\left( \delta_{xx} + \delta_{yy} \right) \Psi^{n+1} = -\omega^{n+1}$$

3 Repeat for next time step

$$\omega(x,1) = -\frac{2}{\Delta_y^2} \left( \Psi(x,1 - \Delta_y) + U(t)\Delta_y \right) + O(\Delta_y)$$





# Adjoint-Variable Approach

Linear, time invariant

$$\begin{array}{l}
 n = 1 \\
 n = 2 \\
 n = 3
 \end{array}
 \begin{bmatrix}
 \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \\
 \begin{bmatrix} -\mathbf{I}_i + \mathbf{G}_W^{n=1} & \textcircled{-\mathbf{S} + \mathbf{G}_P^{n=1}} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\
 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}
 \end{bmatrix}
 \begin{bmatrix}
 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\
 \textcircled{\begin{bmatrix} \mathbf{L}_\omega & \mathbf{0} \\ \mathbf{I}_i & \mathbf{L}_\Psi \end{bmatrix}} \\
 \begin{bmatrix} -\mathbf{I}_i + \mathbf{G}_W^{n=2} & -\mathbf{S} + \textcircled{\mathbf{G}_P^{n=2}} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}
 \end{bmatrix}
 \begin{bmatrix}
 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\
 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\
 \begin{bmatrix} \mathbf{L}_\omega & \mathbf{0} \\ \mathbf{I}_i & \mathbf{L}_\Psi \end{bmatrix}
 \end{bmatrix}
 \Delta \mathbf{X}_{\text{mon}} = -\mathbf{F}_{\text{mon}}$$

Reverse-time
Vorticity BC coupling terms
Jacobians arising from convective terms [apply data compression]

$$\begin{array}{l}
 \uparrow \\
 \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \\
 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\
 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\
 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}
 \end{array}
 \begin{array}{l}
 \begin{bmatrix} -\mathbf{I}_i + \mathbf{G}_W^{n=1} & -\mathbf{S} + \mathbf{G}_P^{n=1} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}^T \\
 \begin{bmatrix} \mathbf{L}_\omega & \mathbf{0} \\ \mathbf{I}_i & \mathbf{L}_\Psi \end{bmatrix}^T \\
 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}^T \\
 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}^T
 \end{array}
 \begin{array}{l}
 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\
 \begin{bmatrix} -\mathbf{I}_i + \mathbf{G}_W^{n=2} & -\mathbf{S} + \textcircled{\mathbf{G}_P^{n=2}} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}^T \\
 \begin{bmatrix} \mathbf{L}_\omega & \mathbf{0} \\ \mathbf{I}_i & \mathbf{L}_\Psi \end{bmatrix}^T
 \end{array}
 \mathbf{C}_{\text{mon}} = \left( \frac{\partial \mathbf{H}}{\partial \mathbf{X}_{\text{mon}}} \right)^T$$

$\mathbf{X}_{\text{mon}}^*$ 
Linearization

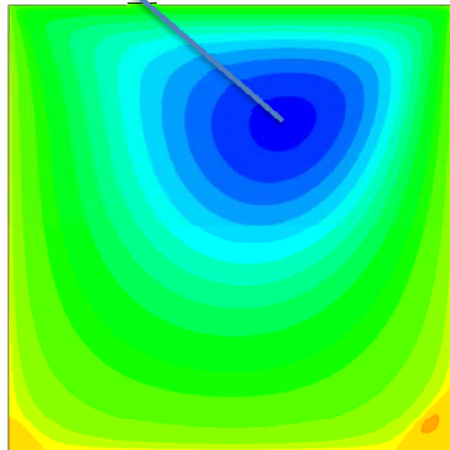




# Verification (Steady State)



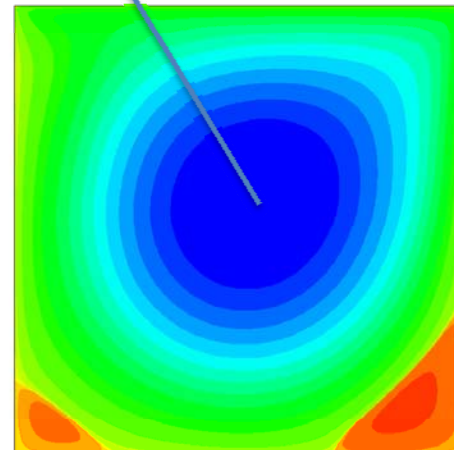
$\Psi_{\min}$  {  $[-.1026, -.1035]$  Collected\*  
                   $-.1035$  Current



$t_{\text{final}} = 20$ , time step = 0.001

$Re = 100$

$\Psi_{\min}$  {  $[-.1163, -.1188]$  Collected\*  
                   $-.1180$  Current



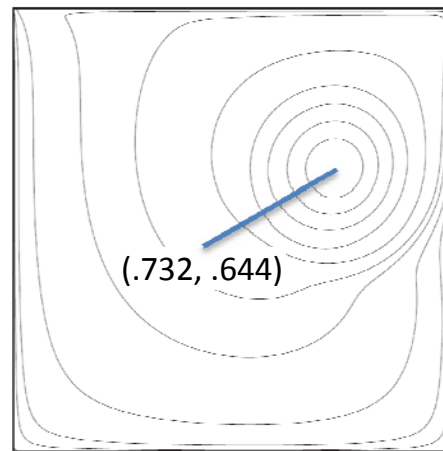
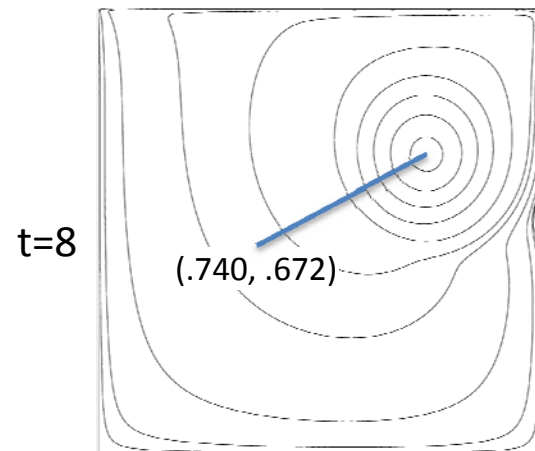
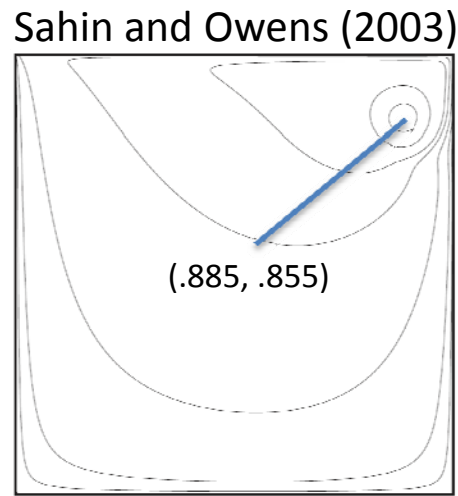
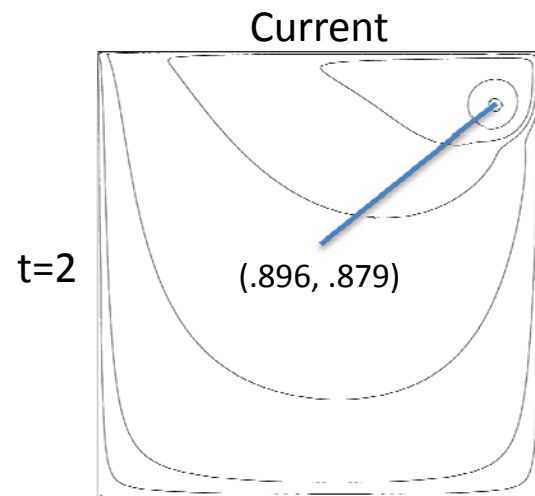
$t_{\text{final}} = 50$ , time step = 0.001

$Re = 1000$

\*Sahin and Owens, "A Novel Fully Implicit Finite Volume Method Applied to the Driven Cavity Problem – Part I: High Reynolds Number Flow Calculations," *Int J Num Methods Fluids*, Vol. 42, Issue 1, May 2003, pp. 79-88



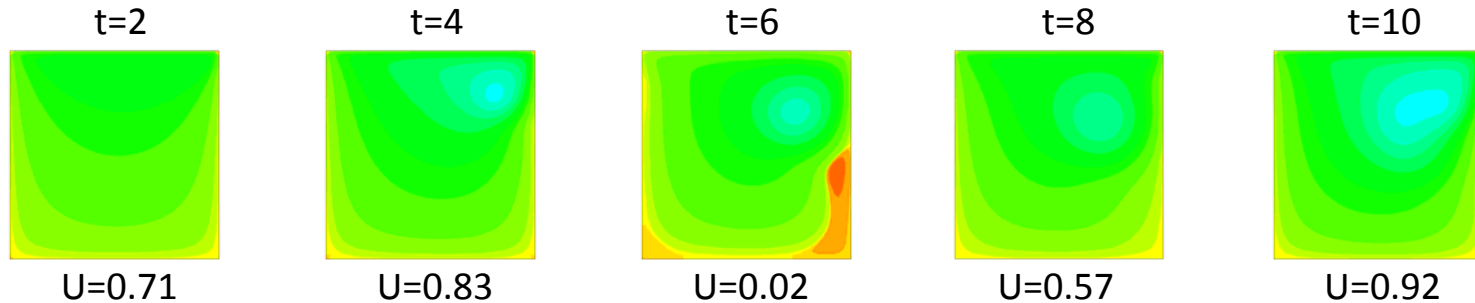
# Verification (Transient)



- $Re=10000$
- $U(t) = 1$
- Contour plots of  $\Psi$
- $t=2$ : agree within 2.8%
- $t=8$ : agree within 4.4%
- Need to explore mesh and time step refinements



# Verification (Sensitivity)



- Re=1000 with baseline mesh ( $101 \times 101$ )
- U varies in time
- Determine sensitivity of  $H_2$  about  $f = 1$
- $H_2$  evaluated at  $t = 10$
- Finite-difference sensitivity:  $\delta f = 0.0001$
- *Sensitivities match to 6 significant digits*

$$U(t) = \frac{1}{2}(1 - \cos(ft))$$

$$H_2(\mathbf{x}_{\text{mon}}) = \sum_k (\Psi_k^n)^2$$

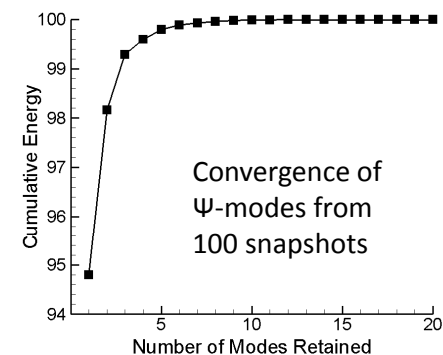
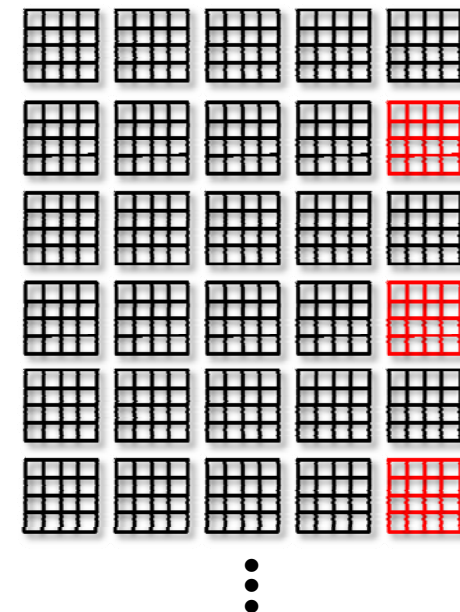
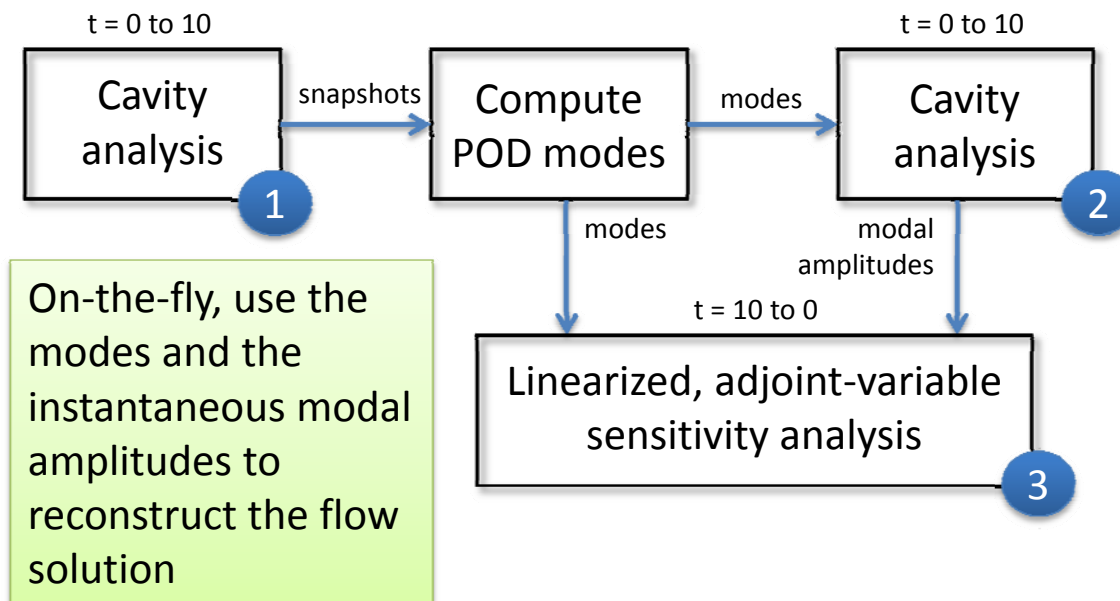
$\partial H_2 / \partial f$ (Adjoint)	$\partial H_2 / \partial f$ (Finite Difference)
4.70771958780	4.7077182309



# POD Data Compression for Sensitivity Analysis

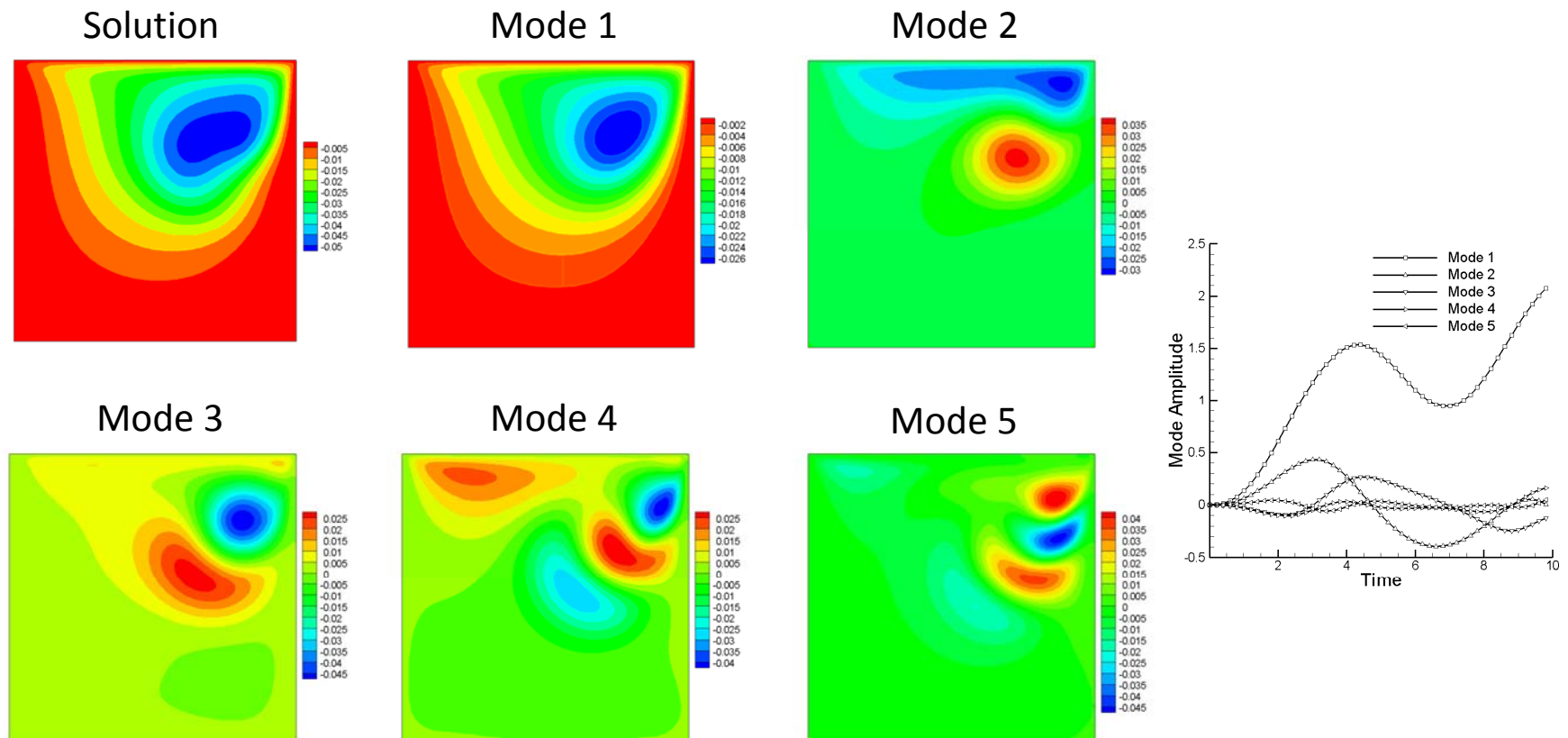


- Same conditions as verification case
- Integration time of 10; 1000 time steps
- Collect snapshots once every 10 time steps
- Decimate snapshot set to coarsen
- Evaluate efficiency and accuracy of POD-based adjoint sensitivity analysis as function of number of snapshots and modes





# Solution and POD Modes (Streamfunction)





# Efficiency and Accuracy

$\partial H_2 / \partial f$  using 100 snapshots

Full order	50 modes	20 modes	10 modes	5 modes
4.707719587	4.707725353	4.711403732	4.724862963	3.121007234

% Error in  $\partial H_2 / \partial f$

	50 modes	20 modes	10 modes	5 modes
100 snapshots	0.00012	0.078	0.36	-34
20 snapshots	–	1.4	3.2	-31
10 snapshots	–	–	1.6	2.8

20 snapshots = 2% of time-history data

10 modes = 1% of time-history data

High efficiency

Good accuracy

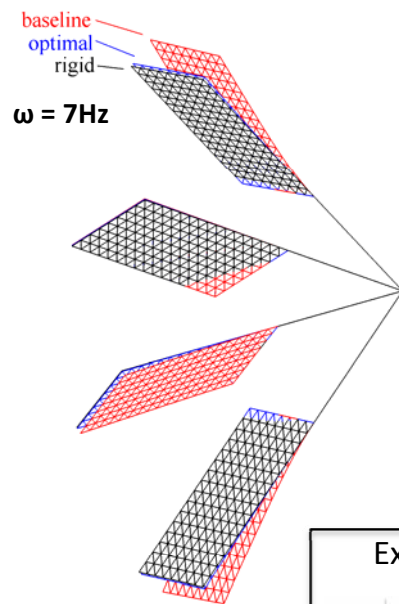
Greatly decrease memory requirement at 2× cost: explore other POD uses



# Structural Design (Inertial Loads Only)

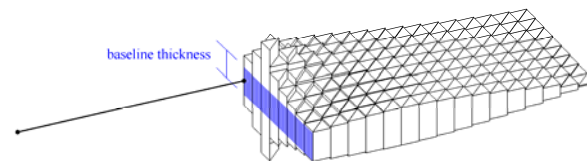


Goal: study transient sensitivity analysis in context of DOF reduction

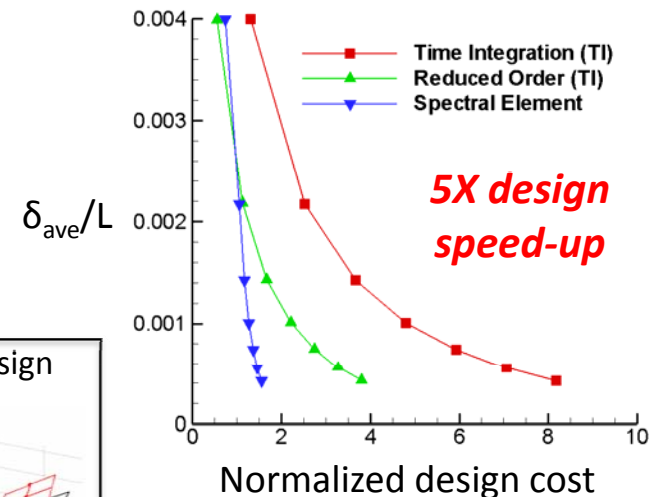
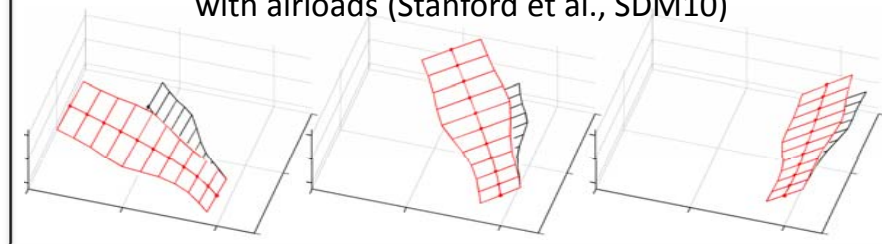


- Identify best thickness distribution for rapidly actuated plate
- Nonlinear modeling of a flapping plate
- Minimize  $\delta_{ave}$  = time-averaged  $\delta$
- 256 variables (element thicknesses w/ constraints)
- GBO via MATLAB (*fmincon*)

Resultant thickness distribution



Extension: Kinematic/Structural design for wing design with airloads (Stanford et al., SDM10)



ROM Adjoints  $\approx$  Free

Stanford, Beran, and Kurdi, "Adjoint Sensitivities of Time-Periodic Nonlinear Structural Dynamics via Model Reduction," *Computers and Structures* (to appear), 2010.

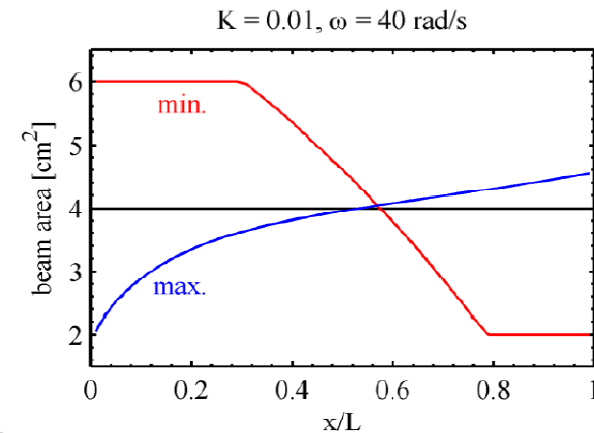
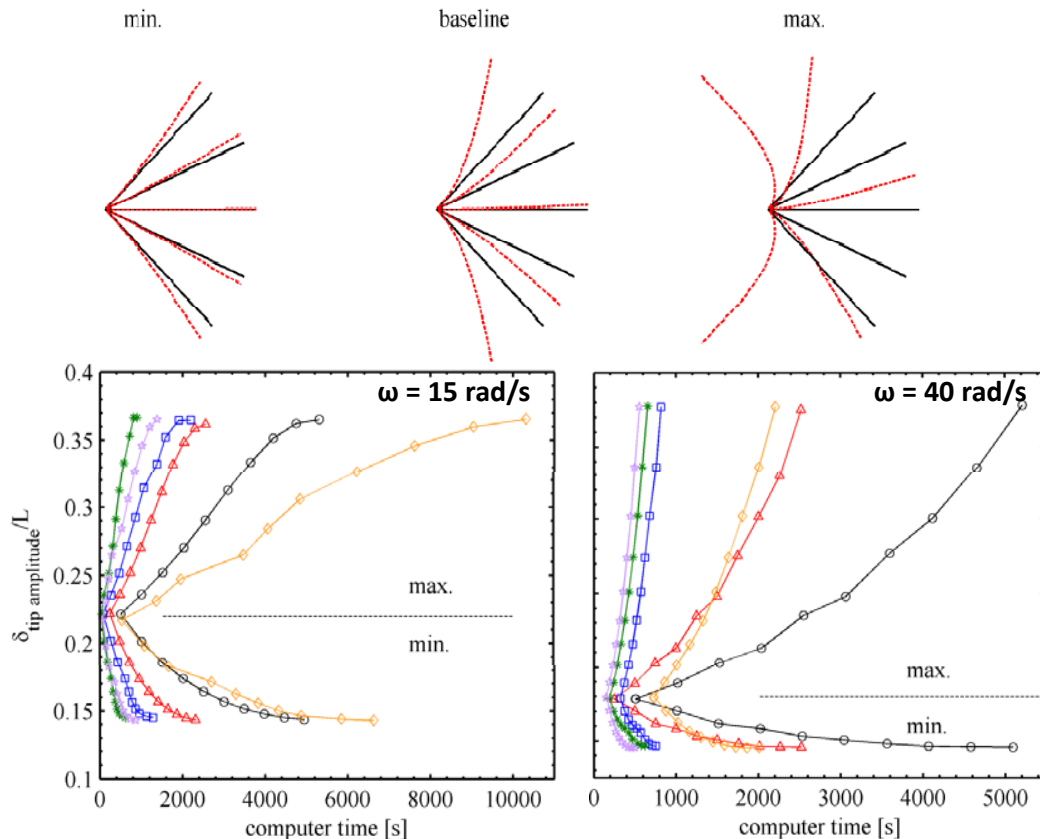




# Beam Design (Inertial Loads Only)



- Identify best area distributions for minimum and maximum time-averaged tip displaced
- Co-rotational FEA formulation; 50 beam elements, each with a different sectional area
- Side constraints on area; GBO via MATLAB (*fmincon*)
- Compute sensitivities with the adjoint formulation



- Newmark, FOM
- △— Newmark, ROM
- SE, FOM
- ◇— FD, FOM
- \*— SE, ROM
- ☆— FD, ROM

Some benefits of combining SE and ROM in time-periodic formulation



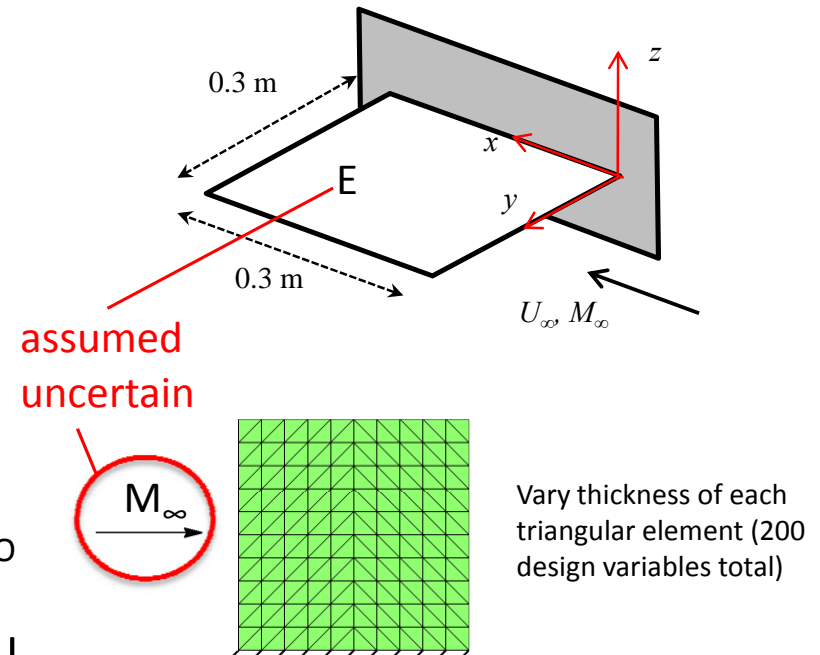
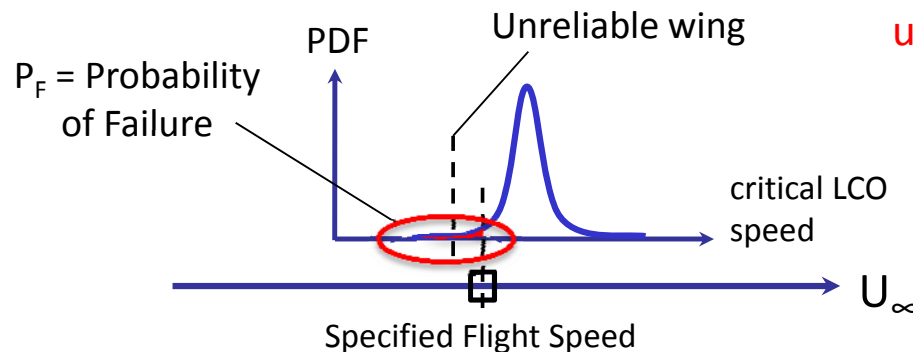


# Reliability-Based Design Optimization (RBDO)



Goal: Examine use of transient sensitivity analysis to design a plate wing that is both light and reliable

- Reliable: wing does not exhibit too severe a limit-cycle oscillation
- $U_\infty > U_{\text{flutter}} \rightarrow$  limit cycle oscillation
- Piston theory aerodynamics ( $M_\infty > 1$ )
- Nonlinear von Kármán plate FEA



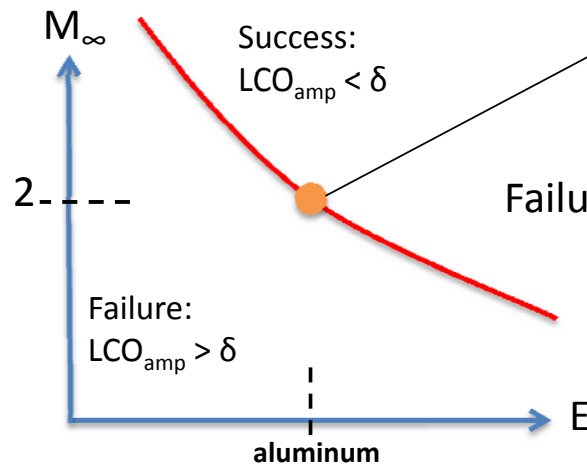
Vary thickness of each triangular element (200 design variables total)

Minimize mass of plate; constrain the probability that  $LCO_{\text{amp}} > \delta$  ( $P_F \leq \sigma$ )



# Contrasting Approaches

## Deterministic Optimization



Generally, the designed plate “moves” to the constraint boundary ( $P_F \approx 1/2$ )

Failure surface:

$$g = g(\mathbf{x}(\mathbf{d}, E, M_\infty)) = \delta - \text{LCO}_{\text{amp}} = 0$$

$\mathbf{x}$  = response variables

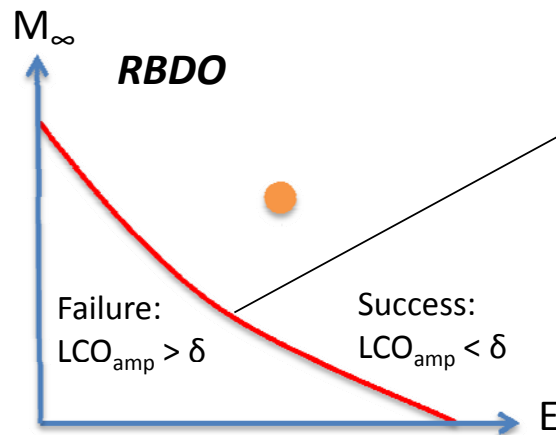
$\mathbf{d}$  = design variables

$$\min_{\mathbf{d}} \text{ weight} = f(\mathbf{d})$$

subject to:

$$g(\mathbf{x}(\mathbf{d}, E, M_\infty)) > 0;$$

side constraints on  $\mathbf{d}$



Generally, the designed plate “moves” away from the constraint boundary a “safe” distance ( $P_F = \sigma$ )

$$\min_{\mathbf{d}} \text{ weight} = f(\mathbf{d})$$

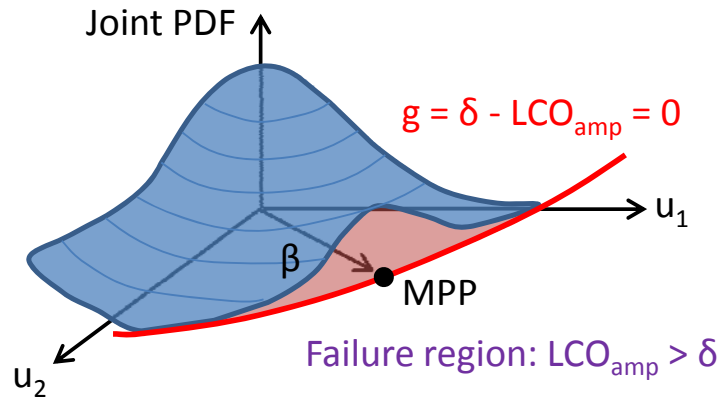
subject to:

$$1 - \text{Prob}(g < 0)/\sigma \geq 0; \text{ side constraints on } \mathbf{d}$$

Allen and Maute, “Reliability-based design optimization of aeroelastic structures,” *Structural and Multidisciplinary Optimization*, Vol. 27, 2004, pp. 228-242. *(Static Aeroelasticity)*



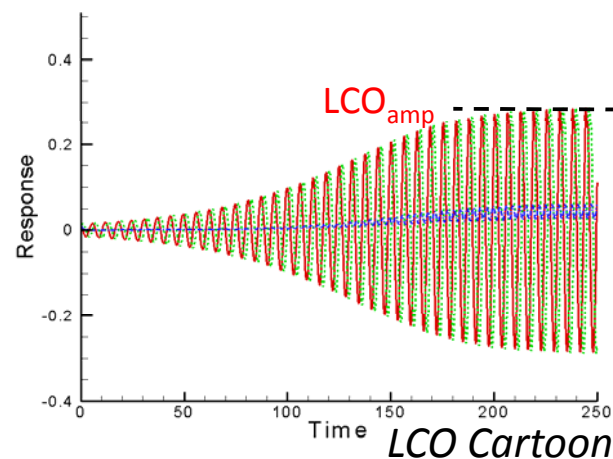
# RBDO Formulation



AIAA Short Course: Introduction to Non-Deterministic Approaches

- $M_\infty$  and  $E$  are chosen to be uncertain (normal)
- Map to uncorrelated random variables  $u_1$  and  $u_2$  in standard normal space
- Compute Most Probable Point (MPP) and reliability index  $\beta$
- Approximate failure surface as linear: First Order Reliability Method (FORM)
- Compute probability of failure,  $P_F = P_F(\beta)$
- Meet  $P_F$  constraint using analytical gradients

- 1 For a given structure, compute MPP using gradient based optimization: require sensitivities of  $g$  to  $u_1$  and  $u_2$
- 2 Reduce weight while meeting  $P_F$  constraint using gradient based optimization: require sensitivities of  $P_F$  to  $d_i$  (found from sensitivities of  $g$  to  $d_i$ )

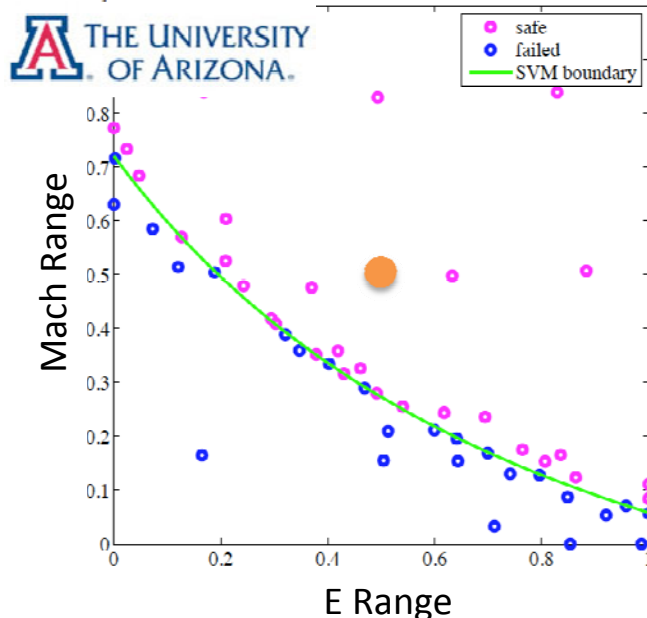


Adjoints of transient solutions used to compute sensitivities of  $g$  to  $d_i$



# RBDO and SVM Results

Uniform (baseline) panel



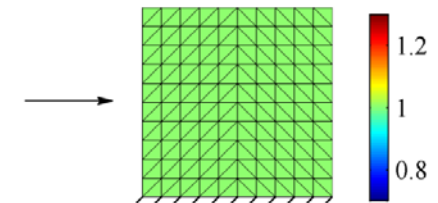
1. Basudhar used Support Vector Machine and adaptive sampling to approximately construct failure surface
2. Computed  $P_F$  with MCS on SVM boundary (55 samples)
3. Computed  $P_F$  with QMCS (Lambe, MSSRC)

Method	PF
FORM	0.0197
MCS ( $10^6$ )	0.0248
QMCS ( $10^4$ )	0.0244

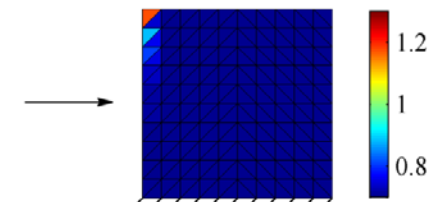
Basudhar and Missoum, "Update of explicit limit state functions constructed using Support Vector Machines," AIAA 2007-1872, April 2007.

RBDO Step	Cost (MATLAB, single CPU)
Simulation	10 minutes
Adjoint	5 minutes
MPP	1 hour
Optimization	4 hours (deterministic), 12 hours (probabilistic)

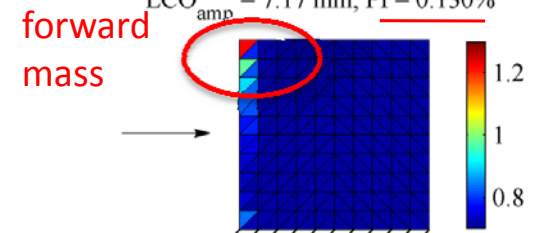
baseline: mass = 0.252 kg  
LCO<sub>amp</sub> = 5.98 mm, Pf = 0.114%



deterministic optimum: mass = 0.178 kg  
LCO<sub>amp</sub> = 10.01 mm, Pf = 50.17%



probabalistic optimum: mass = 0.182 kg  
LCO<sub>amp</sub> = 7.17 mm, Pf = 0.130%





# Recent Activities: Rigid-Body MAV Motions



- Start to investigate impact of rigid-body motion on MAV performance
- Prof. Haibo Dong (WSU), Mr. Zachary Gaston (WSU)
- Mr. Tim Broering (UL)



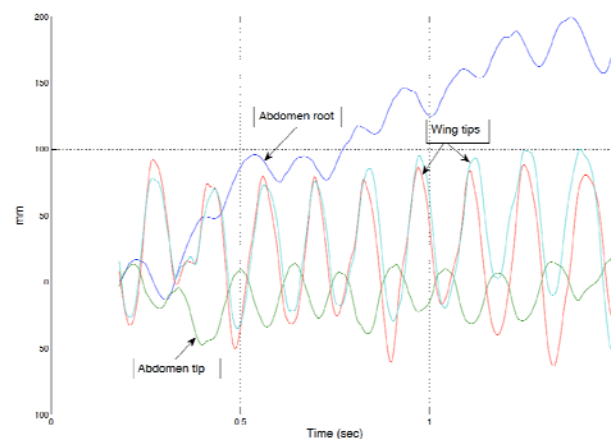
Chakravarthy, Albertani, Evers, "In-Flight Dynamically Adaptive Configurations: Lessons from Live Lepidoptera," AIAA 2010-2828, April 2010.



Digital Image Correlation by Prof. Albertani using live specimens of *Lepidoptera*



McGuire Center for Lepidoptera and Biodiversity, Gainesville, FL



Need to include rigid-body motions and body flexibility in bio-inspired MAV models



# Plan for Rigid-Body Coupling

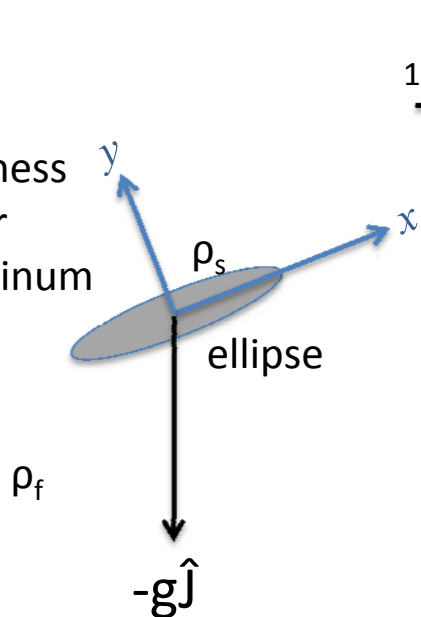


- Emphasize passive motions first: falling bodies in quiescent flow
  - Pesavento and Wang, “Falling Paper: Navier-Stokes Solutions, Model of Fluid Forces, and Center of Mass Elevation”, *PRL*, Vol. 93, No. 14, 2004
  - Modify high-fidelity tools to repeat 2D simulations and extend in 3D; validate at WSU with high-speed photography (want comparisons)
  - Calibrate quasi-steady models (like those used in flapping)
- Examine influences of gust and variability on falling motions
  - Introduce variability into quasi-steady models (e.g., how is seed dispersal impacted by winds?)
- Re-examine design procedures that have been developed so far: want MAVs that are robust to gust

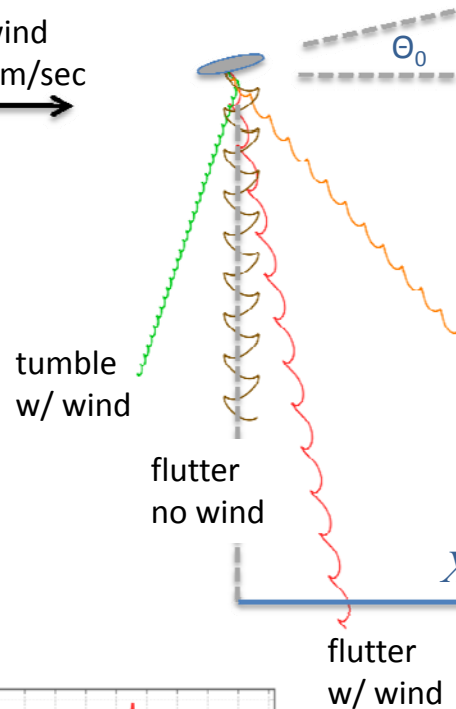


# Some Typical Motions

$c$  = chord  
 $h$  = thickness  
 $\rho_f$  = water  
 $\rho_s$  = aluminum



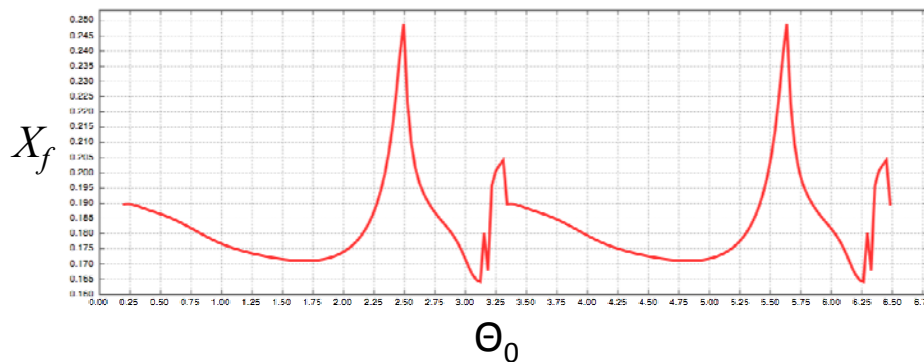
wind  
 10 cm/sec



Trajectories computed with  
 quasi-steady aerodynamics  
 (Wang et al.)

flutter:  $\beta = h/c = 1/14$   
 tumble:  $\beta = h/c = 1/5$

tumble  
 no wind  
 (descent speed =  
 11.5 cm/sec)



- Will explore impact of variability (physical and model) on distribution of landing locations
- Developed transient sensitivities (direct): role in selecting bodies with more desirable falling characteristics?

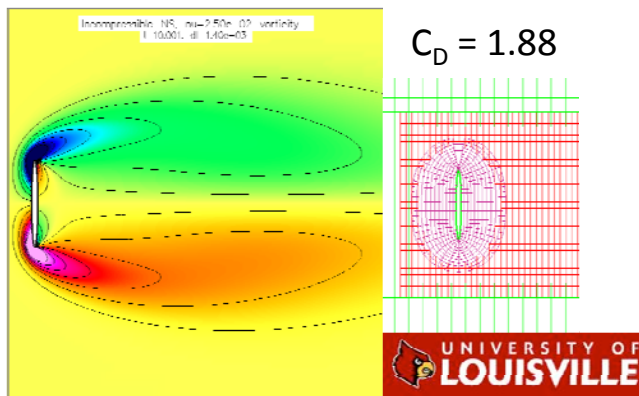




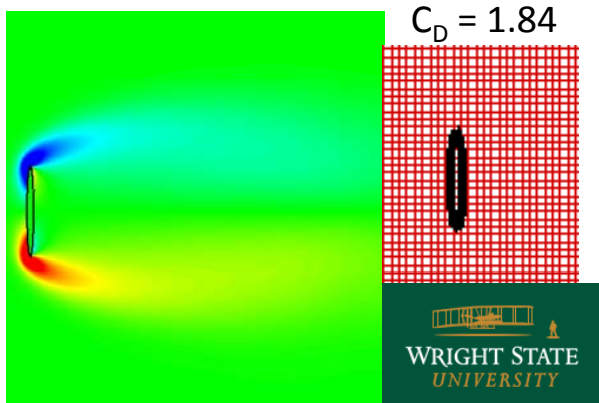
# High-Fidelity Results

$Re = 40$  (Stationary)

Overture

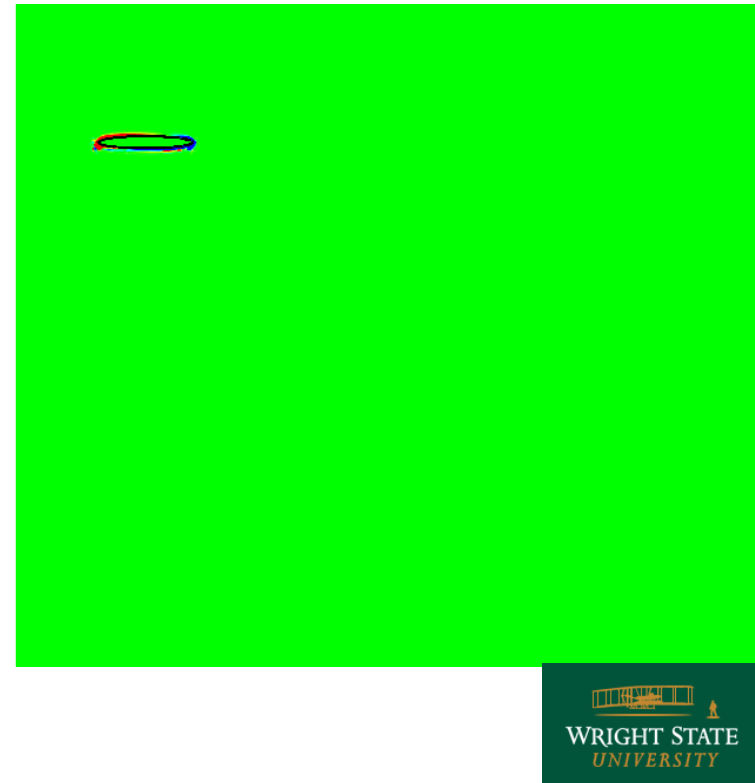


VICAR3D



$Re = O[10^3]$  (Falling)

Preliminary VICAR3D result







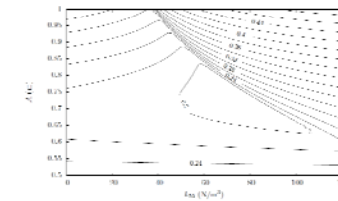
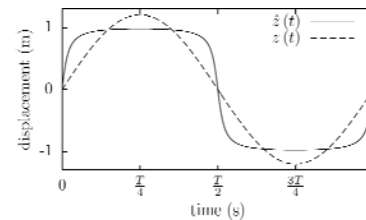
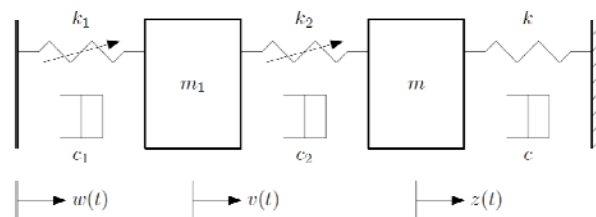
# Recent Activities (cont.)



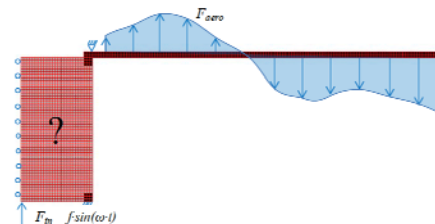
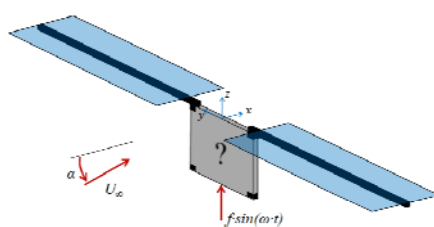
McFarland and Hubbard



- Start to explore role of actuation mechanism in MAV design
  - Investigate physical interactions between a flapping wing and the mechanism that flaps the wing (e.g., transmission of inertial loads)



- Developing compliant mechanisms via topological optimization
  - Link mechanism with generated inertial/aero loads (MAO 2010)



Understanding/modeling energy transfers between mechanism and wing critical



# Concluding Remarks

- Sensitivity analysis of transient/time-periodic systems serves an important role for design of both large and small aircraft
  - Constraint boundaries often nonlinear (LCO and aeroelastic response in gust); strive for physics-based approaches not reliant on safety factors
  - Essential for design of flapping wing MAVs; strive for physics-based approaches that account for gust
- Lessons learned through unsteady sample problems
  - POD is a straightforward means for data compression in sensitivity analysis for large systems; extensions using POD ripe for study
  - Adjoint vectors in ROM formulation computed virtually for free (tailoring of structure for nonlinear response during rotary actuation)
  - Adjoint-based sensitivities work well in an RBDO context; want to extend (e.g., transonic, SVM, SORM) based on lessons learned
- Interesting departure points for further study: variability in motion subject to gust, mechanism design



# Recent Publications

- Stanford, B., and Beran, P., "Adjoint Sensitivities of Time-Periodic Nonlinear Structural Dynamics via Model Reduction," *Computers and Structures* (to appear), 2010.
- Stanford, B., and Beran, P., "Analytical Sensitivity Analysis of an Unsteady Vortex Lattice Method for Flapping Wing Optimization," *Journal of Aircraft*, Vol. 47, No. 2, Mar.-Apr. 2010, pp. 647-662.
- Ghommem, M., Hajj, M.R., Pettit, C.L., and Beran, P.S., "Stochastic Modeling of Incident Gust Effects on Aerodynamic Lift," *Journal of Aircraft* (to appear), 2010.
- Missoum, S., Dribusch, C., and Beran, P., "Reliability-Based Design Optimization of Nonlinear Aeroelasticity Problems," *Journal of Aircraft*, Vol. 47, No. 3, May-June, 2010, pp. 992-998.
- Kurdi, M., Beran, P., Stanford, B., and Snyder R., "Optimal actuation of nonlinear resonant systems," *Structural and Multidisciplinary Optimization*, Vol. 41, No. 1, Feb. 2010, pp 65-86.
- Pettit, C.L., Hajj, M.R., and Beran, P.S., "A Stochastic Approach for Modeling Incident Gust Effects on Flow Quantities," *Probabilistic Engineering Mechanics*, Vol. 25, Issue 1, Jan. 2010, pp. 153-162.
- Stanford, B., Beran, P., and Kurdi, M., "Model Reduction Strategies for Nonlinear Beams Subjected to Large Rotary Actuations," *Aeronautical Journal*, Vol. 113, No. 1150, Dec. 2009, pp. 751-762.



# Questions?





# AMP Team Composition (WPAFB)



**Mission: Integrate multiple disciplines to *discover and exploit new phenomena* for system *optimization and assessment* of revolutionary aerospace vehicles**

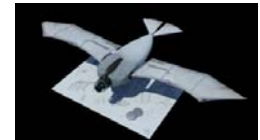


Center Director

Branch Chief  
- Tech Advisor

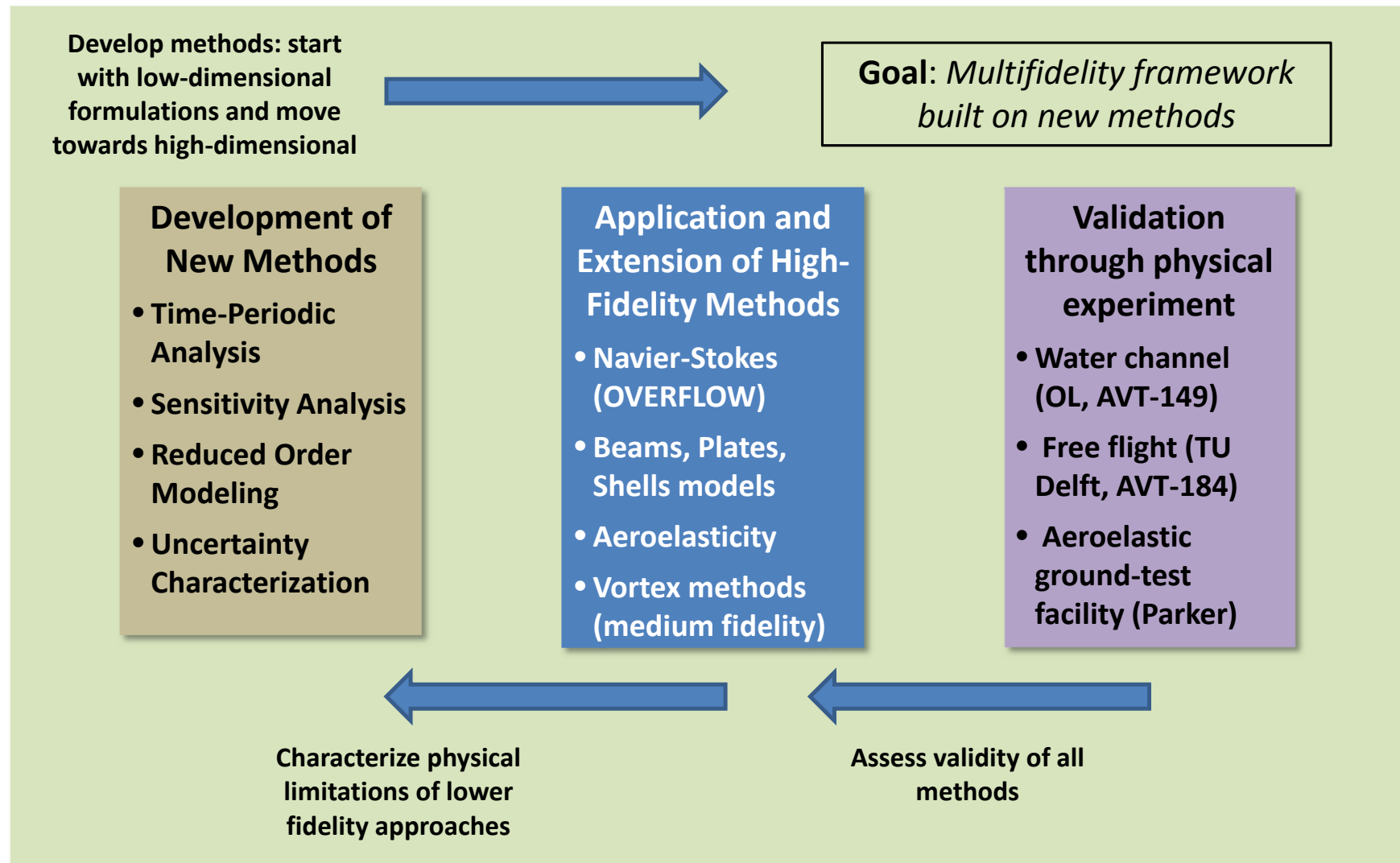
## Analysis Methods for Prototypes

- Dr. José Camberos – On detail as RB Deputy Chief Scientist
- Dr. Chris Chabalko – Postdoc (NRC, UTC)
- Dr. Ned Lindsley – Supporting prototype validation/assessment
- Dr. Aaron McClung – Civil Servant, formerly NRC
- Mr. John Moore – Undergraduate Co-op (University Florida)
- Mr. Michael Robbeloth – Computer Scientist, DSA
- Dr. Rich Snyder
- Dr. Bret Stanford – Postdoc (NRC)
- Dr. Phil Beran - Lead





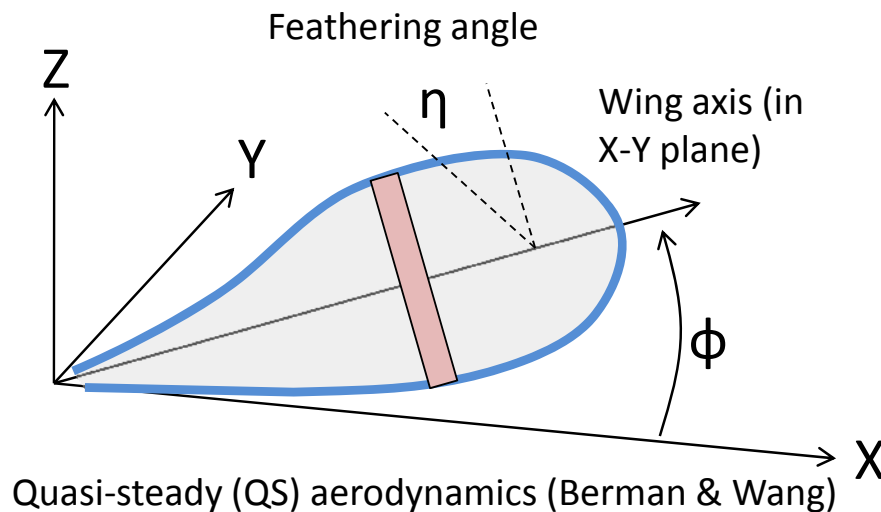
# Methods Development Strategy





# Application to Insect Wing

Berman and Wang, "Energy-Minimizing Kinematics in Hovering Insect Flight," *JFM*, Vol. 582, 2007 (Rigid wing with stroke-plane deviations)

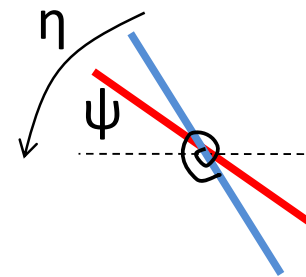


Power reduction from initial design:

- 55% for unconstrained acceleration
- 40% for constrained acceleration

*Looking at inertial power contribution*

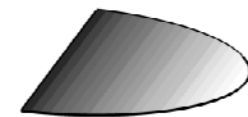
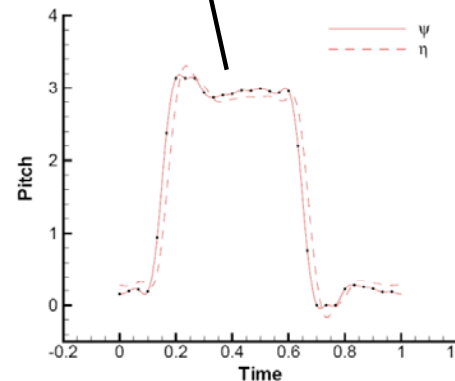
Kurdi, Beran, Stanford, and Snyder, "Optimal Actuation of Nonlinear Resonant Systems," *Structural and Multidisciplinary Optimization*, Published online June 2009.



Prescribed ( $\psi$ ) and realized ( $\eta$ ) angles

- mass-spring-damper
- inertial & aero loads

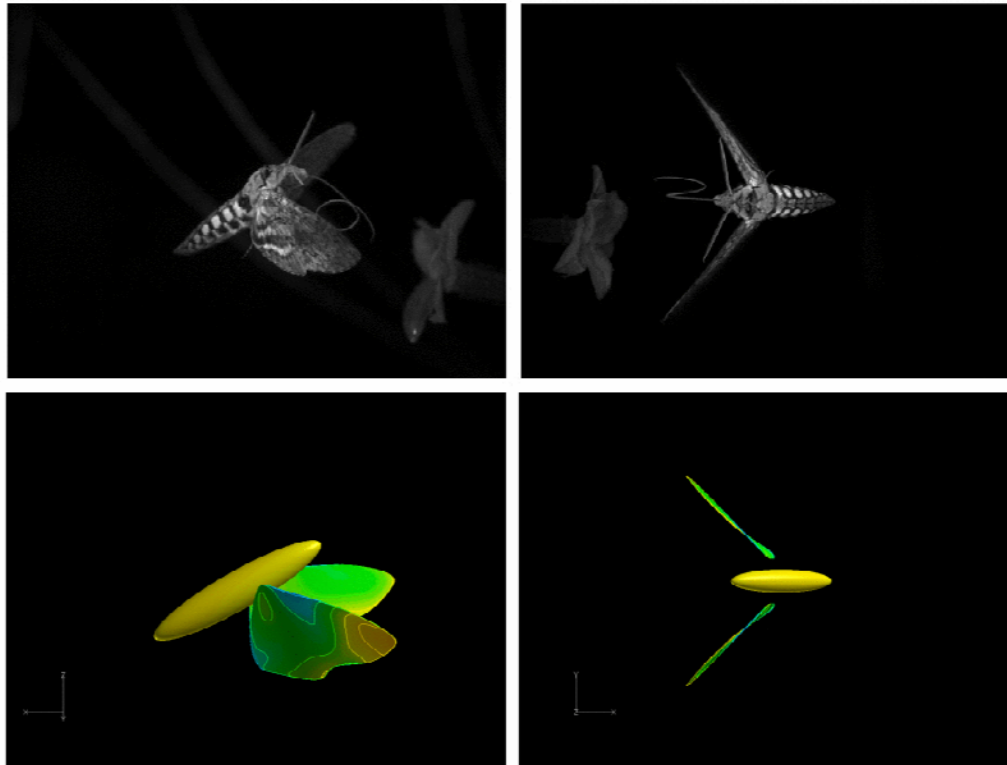
Large snap rotations favored



Optimized fruitfly wing kinematics (235 Hz)



# High-Fidelity Analysis



## Understanding Complex Physics



- Study Hawkmoth physics using Navier-Stokes (NS) simulation
- Collaboration with AFIT
- Hawkmoth kinematics (hover)
- What's new?
  - OVERFLOW 2.1 Elastic (5<sup>th</sup>/2<sup>nd</sup>-order in space/time)
  - Prescribed wing deformations
  - Variations in kinematics
- *Moderate flexibility increases hover efficiency*

Planform	Study	Fx (N)	Fy (N)	Fz (N)
Rectangular	Current Work	7.24e-04	-1.46e-03	7.26e-03
<i>Manduca sexta</i>	Current Work	8.42e-04	-1.65e-03	6.16e-03
<i>Agrius convolvuli</i>	Aono and Liu [1]	1.20e-03	-1.20e-03	8.48e-03

McClung, "Influence of Structural Flexibility on Flapping Wing Propulsion," AFIT Dissertation, April 2009